Microwave Measurement of High-Dielectric-Constant Materials

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Abstract—A particularly severe source of error in the microwave measurement of high dielectric constants has been the presence of small air gaps between dielectric surfaces and metal walls. In this paper, two precise measurement techniques are described that eliminate the effect of air gaps through the use of resonant modes, for which $E_n = 0$ at the dielectric-to-metal interface. In one method a cylindrical sample is resonated within a closely fitting circular waveguide, and in the other a cylindrical sample is placed at the center of a radial waveguide. Both methods utilize a circular-electric-mode whose electric field is parallel to the metal walls. The waveguides are dimensioned to be cut off in their air regions at the resonant frequency of the dielectric sample. Formulas yield ϵ_r as a function of the resonant frequency, and of diameter and length of the dielectric cylinder. Measured data on samples having $\epsilon_r \approx 85$ show the two methods to agree within a few tenths of one percent. The accuracy of the methods is on the order of 0.5 percent maximum error and 0.2 percent probable error when f_0 is measured within 0.1 percent and D and L within 0.0005 inch (for $D \approx 0.3$ inch and $L \approx 0.1$ inch).

I. Introduction

At low frequencies, the determination of dielectric constant is usually based on measurement of capacitance of a parallel-plate or coaxial capacitor containing the material under test. At microwave frequencies, dielectric constant is determined through measurement of the effects of the enhanced capacitance produced by the dielectric material. These effects include a change in velocity of propagation in a dielectric-loaded transmission line, a change in the resonant frequency of a dielectric-loaded resonant cavity, and reflections at the boundaries [1], [2]. The accuracy of the measurement is seriously affected, however, when the configuration permits electric field lines to pass from the dielectric specimen to the conducting boundaries by way of irregular or unknown air gaps. The error due to air gaps is especially severe in the measurement of high-dielectric-constant materials.

Figure 1 illustrates the parallel-plate-capacitor case, in which the electric field passes through the dielectric material and the air gap. Air gaps of average thickness t_a will exist in practice, unless all mating surfaces are perfectly smooth and flat, or unless a conductive coating is plated directly on the dielectric sample. In the presence of air gaps, the effective capacitance of the series combination of dielectric capacitor and air capacitor is proportional to

Manuscript received December 28, 1965; revised May 9, 1966. The work described in this paper was supported by the U. S. Army Electronics Labs., Fort Monmouth, N. J., under Contract DA 36-039-AMC-02267(E), "Investigation of Microwave Filters Utilizing Dielectric Resonators."

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$$C_{\text{total}} \propto \frac{\epsilon_r}{t_d + 2t_a\epsilon_r} = \frac{\epsilon_r'}{t_d} \tag{1}$$

where ϵ_r is the true value of relative dielectric constant and ϵ_r' is the apparent value of relative dielectric constant yielding the same C_{total} with t_a neglected. When solved for ϵ_r', (1) becomes

$$\epsilon_r' = \frac{\epsilon_r}{1 + \dfrac{2t_a\epsilon_r}{t_d}} . \tag{2}$$

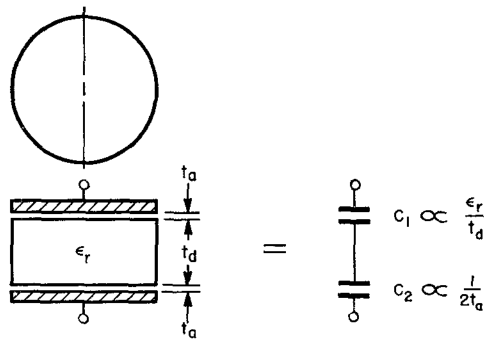

Fig. 1. Dielectric capacitor with air gaps.

As an example, consider t_d to be on the order of 0.1 inch and ϵ_r on the order of 100. Because of surface roughness, imperfectly parallel surfaces, etc., t_a may be expected to approach 0.0001 inch despite considerable care. Substitution in (2) yields a ratio of apparent to true ϵ_r on the order of 0.833. Thus, a dielectric constant error of 17 percent results from the presence of difficult-to-avoid air gaps as small as 0.0001 inch. Small air gaps can affect accuracy in a similar way when dielectric samples are placed in a resonator or transmission line.

This paper describes two methods by which the air-gap problem may be avoided at microwave frequencies. In both cases, resonator configurations that operate in modes having $E_n = 0$ on the surfaces of the dielectric sample adjacent to metallic walls are utilized. Remembering that E_t is always nearly zero close to a metal wall, we see that the stored electric energy virtually vanishes in the air gap. Consequently, the effect of a small air gap on the measurement is negligible. In the first measurement method, a sample in the shape of a right-circular cylinder is resonated within a close-fitting circular waveguide which is below cutoff in its air-filled regions. The second method is similar except that a radial waveguide is used. (The radial waveguide technique has been previously used by Hakki and Coleman [3] with examples of data in the range $\epsilon_r = 2$ to 3.)

The measurement of loss tangent is not treated in this paper, although the techniques may be extended to yield this quantity in addition to ϵ_r. In the case of high-dielectric-constant samples, it is more accurate and convenient to measure the unloaded Q, Q_u, of the sample as a band-stop resonator at the center of a large propagating waveguide [4], [5], [6], [7]. If the electric energy were completely stored inside the sample, and if dissipation loss outside the sample were zero, the loss tangent would be exactly $1/Q_u$. These conditions are not met perfectly in practice, but when ϵ_r is greater than 50, virtually all of the electric energy is stored in the resonant sample, and dissipation loss on the relatively distant waveguide walls is very small. (Magnetic loss is assumed to be zero in this discussion.) Most of the authors' experience has been with samples of TiO_2 ceramic of varying density and impurities. Dielectric constants in the range 80 to 115 and loss tangents of 0.00008 to about 0.0002 have been measured.

II. The Circular Waveguide Dielectrometer

Figure 2 illustrates the geometry of the circular waveguide dielectrometer. In any given waveguide mode, resonances of the dielectric sample can occur when D/λ is a value such that the mode is above its cutoff frequency in the dielectric region and below its cutoff frequency in the air regions. Because of the exponential decay of the fields in the air regions, the total stored energy of the resonance is mainly within the dielectric sample. Resonance occurs when the following condition is satisfied.

$$B_a + B_d = 0 \tag{3}$$

where B_a and B_d are the susceptances at one of the transverse surfaces of the sample looking toward the air region and dielectric region, respectively. The lowest-order resonant mode for which the normal component of electric field E_n is zero over the surface of the dielectric, as required to make air-gap effects negligible, is the TE_{011}. (The first two subscript integers denote the waveguide mode, while the third integer denotes the order of resonance in an increasing set of discrete resonant lengths.) The characteristic admittance Y_0 of the waveguide is real in the propagating dielectric region and imaginary in the cutoff air region. The susceptance B_a is equal to Y_0/j of the air-filled waveguide, as follows for the TE_{01} mode:

$$B_a = \frac{Y_{0a}}{j} = -\frac{1}{\eta}\left(\left(\frac{\lambda}{0.820D}\right)^2 - 1\right)^{1/2} \tag{4}$$

where $\eta = 376.7$ ohms is the characteristic impedance of free space, λ is free-space wavelength, and D is the diameter of the waveguide and sample. Equation (4) implies that the cutoff waveguide extends to plus and minus infinity. In practice, insignificant error occurs when the

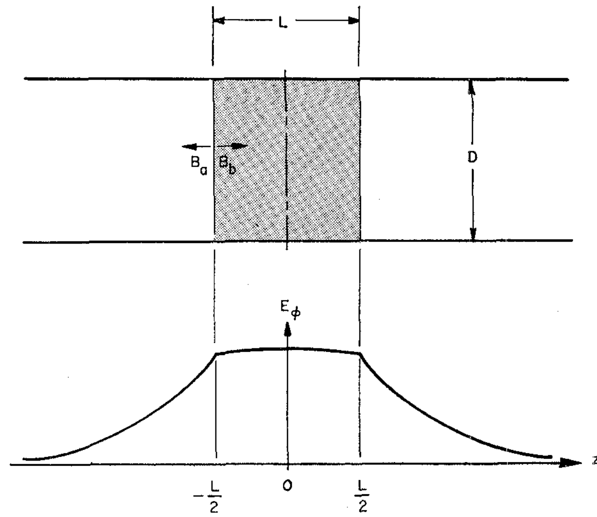

Fig. 2. Resonant electric field distribution in the circular waveguide dielectrometer, TE_{011} mode.

air-filled waveguide is terminated at points at which the evanescing wave is attenuated by at least 30 dB.

In the TE_{011} mode of resonance, the transverse field E_ϕ is a maximum at $z=0$, as shown in Fig. 2, while $H_t=0$ at $z=0$. Thus a magnetic wall, or open-circuiting plane, may be assumed to exist at $z=0$. The susceptance B_d is therefore

$$B_d = Y_{od} \tan\frac{\beta L}{2} = \frac{\lambda}{\eta\lambda_{gd}} \tan\frac{\pi L}{\lambda_{gd}} \tag{5}$$

where

$$\lambda_{gd} = \frac{\lambda}{\left(\epsilon_r - \left(\frac{\lambda}{0.820D}\right)^2\right)^{1/2}}. \tag{6}$$

When (4) through (6) are substituted into (3), we obtained:

$$\left(\epsilon_r - \left(\frac{\lambda}{0.820D}\right)^2\right)^{1/2} \tan\left[\frac{\pi L}{\lambda}\left(\epsilon_r - \left(\frac{\lambda}{0.820D}\right)^2\right)^{1/2}\right] = \left(\left(\frac{\lambda}{0.820D}\right)^2 - 1\right)^{1/2}. \tag{7}$$

Equation (7) is a transcendental equation which may be solved for ϵ_r as a function of directly measurable dimensions and the resonant frequency $f = c/\lambda$, where c is the velocity of light in free space. Employment of this technique is straightforward, except that the desired TE_{011} resonance must be identified among the various TE_{111}, TM_{011}, TM_{111}, etc., resonances which also appear. This identification is accomplished very simply in practice. All modes other than circular-electric modes produce longitudinal current on the conducting walls of the circular waveguide. Hence, when the two bored blocks shown in Fig. 3 are separated slightly, the TE_{011} resonance is scarcely affected, while the other resonances are severely detuned. Note that in Fig. 3 the test sample is placed off-center with respect to the gap. This is done to

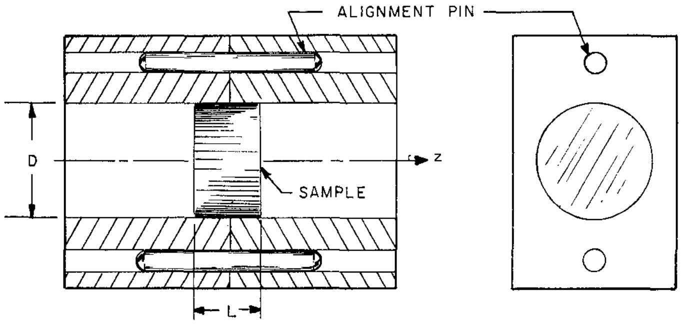

Fig. 3. Split-block structure of the circular waveguide dielectrometer.

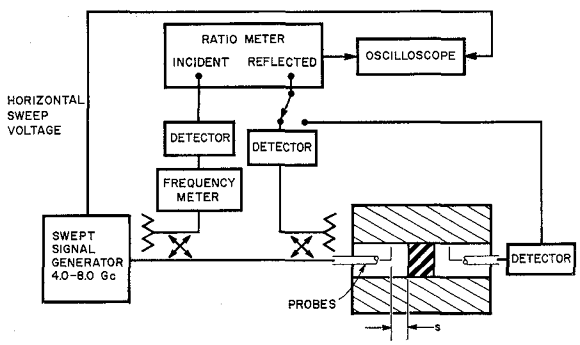

Fig. 4. Laboratory instrumentation for observation of resonances.

prevent improper mode identification, which would result if the gap coincided with the null of longitudinal current at the sample's midplane in TE_{111}, TM_{012}, etc., cases. Higher TE_{0mn} modes are also unaffected by the gap, but their resonant frequencies are separated so widely from the TE_{011} resonance that they would ordinarily present no problem of ambiguity.

Figure 4 illustrates the swept-frequency setup employed to determine the resonant frequencies of samples placed in the circular-waveguide dielectrometer. Either the transmission or reflection response may be utilized for this purpose. The probes are made from small-diameter rigid coaxial line. The exposed center conductor is bent and the coaxial line inserted so that a section of the probe's length is parallel to a desired circular-electric field line. The probe must be placed rather close to the sample in order to achieve sufficient coupling, but not so close as to affect the resonant frequency. Since the dielectric constant of the material to be tested is usually known approximately, the process of finding the TE_{011} mode resonance is aided if a preliminary value of the resonant frequency is computed to direct the operator's attention to the correct portion of the frequency sweep.

An initial test was made with a waveguide diameter of 0.3438 inch, and a polycrystalline TiO_2 sample of diameter 0.3434 inch and length 0.1184 inch. The measured resonant frequency was 5532 Mc/s, and the solution of (4) yielded a value $\epsilon_r = 86.9$. The diameter of this sample was then reduced to 0.3424 inch, thus increasing the diameter difference from 0.0004 to 0.0014 inch. As expected, no change in resonant frequency was observed, and hence the computed ϵ_r value was unaffected.

The proper diameter D to be used in (7) when a small air gap exists is the diameter of the waveguide bore, rather than the diameter of the sample. With this definition of D, a simple application of perturbation theory shows that the relative error in resonant λ and computed ϵ_r is proportional to $(t_a/D)^3$, and hence is very small when t_a/D is small.

III. Radial Waveguide Dielectrometer

The circular waveguide dielectrometer requires a fixed diameter of the sample to be tested. The radial waveguide dielectrometer does not require either a definite diameter or length [3]. As shown in Fig. 5, the configuration consists of a cylindrical cavity with a close-fitting piston at the upper plane surface. The sample is placed at the center of the cavity. The TE_{01} mode in radial waveguide has only an E_ϕ component of electric field, and hence satisfies the desired condition of $E_n = 0$ over the surface of the sample. (The subscript 01 denotes zero variation of the fields from $\phi = 0$ to 2π, and a single maximum in E_ϕ between $z = 0$ and $z = L$. It should be noted that radial waveguide modes are, in general, either TE or TM with respect to the z axis, rather than with respect to the r vector.) In utilizing the device for determining ϵ_r, the cavity height should be less than one-half wavelength in air to ensure that the TE_{01} mode is below cutoff in the air-filled region of Fig. 5, but greater than one-half wavelength in the dielectric sample to permit resonance in the TE_{01} mode.

The resonance condition (3) must be satisfied, where now B_a is the susceptance looking radially into the air region, and B_d is the susceptance looking radially into the dielectric sample. The following resonance condition has been derived by Hakki and Coleman [3].

$$\alpha \frac{J_o(\alpha)}{J_1(\alpha)} = -\beta \frac{K_o(\beta)}{K_1(\beta)} \tag{8}$$

where

$$\alpha = \frac{\pi D}{\lambda}\left(\epsilon_r - \left(\frac{\lambda}{2b}\right)^2\right)^{1/2} \tag{9}$$

$$\beta = \frac{\pi D}{\lambda}\left(\left(\frac{\lambda}{2b}\right)^2 - 1\right)^{1/2} \tag{10}$$

and $J_o(\alpha)$, $J_1(\alpha)$, $K_o(\beta)$, and $K_1(\beta)$ are Bessel functions tabulated in the literature [8]. Equation (8) assumes the radial-wave attenuation from the surface of the sample to the rim of the cavity to be infinite. The error in this assumption is negligible when the radial distance

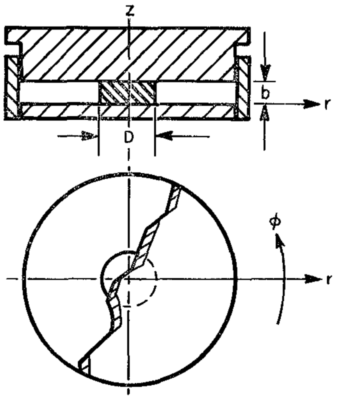

Fig. 5. The radial waveguide dielectrometer.

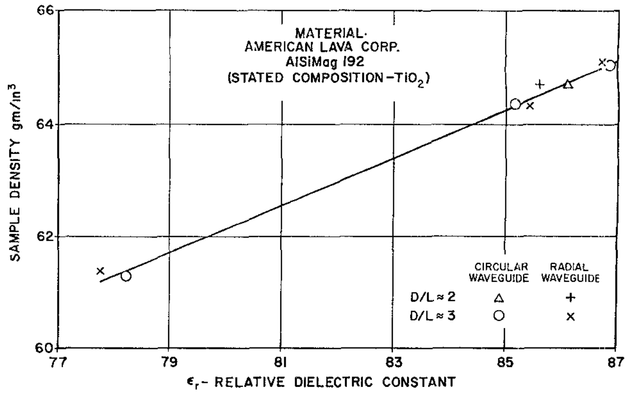

Fig. 6. Measured dielectric-constant values related to sample densities.

from the sample to the rim exceeds $2b$. Careful centering of the sample is also not necessary in this case. (Subject to this assumption, it does not matter whether the circumference of the radial configuration is terminated by a short circuit, as in Fig. 5, or by an open boundary, as in Hakki and Coleman's paper [3].) With D and $b=L$ known by physical measurement, and λ known by electrical measurement, (15) can be solved numerically for α vs. β. Then

$$\epsilon_r = 1 + \left(\frac{\lambda}{\pi D}\right)^2 (\alpha^2 + \beta^2). \tag{11}$$

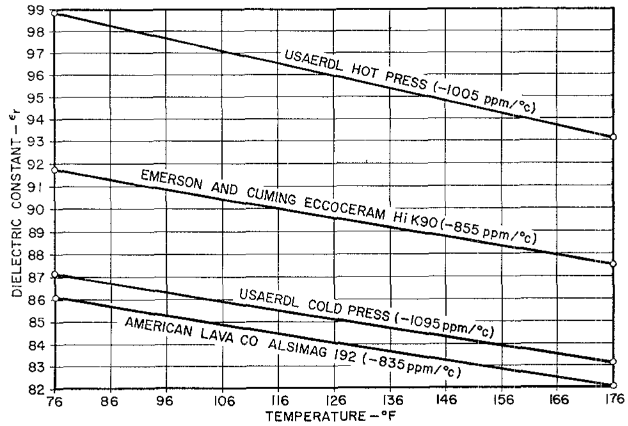

Fig. 7. Dielectric constant plotted vs. temperature, with measured temperature coefficient shown in brackets. (USAEL samples were made at U. S. Army Electronics Labs., Ft. Monmouth, N. J.)

In operation of the radial-waveguide dielectrometer, a simple probe is inserted in the air region to couple as loosely as will allow adequate resonance indication on the oscilloscope. Probe-to-sample spacings on the order of b were found to give good resonance indication with no shifting of the resonant frequency.

The probe couples to a multiplicity of modes. Many resonances are visible on the swept-frequency oscilloscope display. The resonances of all modes having a nonzero E_z component are easily eliminated by noting which resonances move lower in frequency when pressure on the upper plate reduces the minute air gaps due to surface roughness (without measurable change in the plate spacing). The pressure producing the effect is far less than required to mechanically deform the test samples. The same pressure that causes E_z-type mode resonances to move rapidly along the oscilloscope trace has practically no effect on the H_z-type ($E_z=0$) mode resonances. The TE_{011} resonance is the lowest order mode resonance of the H_z type. It is so far removed from the next H_z-type resonance that no difficulty is encountered in identification. This is especially so if attention is directed to the frequency region indicated by a rough calculation of resonant frequency based on the assumed ϵ_r. The approximate dielectric constant is usually known well enough to limit usefully the frequency region to be searched.

IV. Experimental Data

Figure 6 shows a plot of ϵ_r data obtained by both methods on four AiSiMag 192 (American Lava Corp.) samples of varying density. Three of the samples had dimensions $D=0.343$ inch and $L=0.118$ inch, and resonated between 6500 and 7000 Mc/s. The fourth sample had dimensions $D=0.234$ inch and $L=0.116$ inch, and resonated near 7600 Mc/s. The plotted points show a smooth relationship between dielectric constant and density, and good agreement between the two methods.

The effect of temperature on the dielectric constant of TiO_2 ceramic samples is shown in Fig. 7. This data was taken with the radial waveguide dielectrometer placed in an oven. All of the samples are basically TiO_2 ceramic, but their wide range of dielectric-constant values indicates substantial differences in impurities and in processing.

V. Conclusions

The two methods of dielectric-constant measurement described in this paper have been found to be accurate and convenient. They successfully eliminate air-gap error through the use of resonant modes having $E_n=0$ at dielectric surfaces adjacent to metal walls.

The precision of the two methods has been evaluated by allowing f, D, and L to vary in the formulas. For $D\approx 0.3$ inch and $L\approx 0.1$ inch, tolerances of ± 0.1 percent in f, and ± 0.0005 inch in D and L lead to maximum possible errors of about ± 0.5 percent in ϵ_r, and probable errors of about ± 0.2 percent. An improvement of accuracy by at least a factor of 10 is feasible, but is not necessary for most practical purposes.

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The Measurement of Phase at UHF and Microwave Frequencies

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***Abstract*—A theoretical analysis and a unifying classification of methods of measuring phase at UHF and microwave frequencies are presented. The coherent phase bridge circuits are analyzed in terms of the type of modulation applied to the channels of the bridge and the type of combiner and mixer employed at the output of the bridge. In this analysis and classification, identifying characteristics, and some of the relative advantages and disadvantages of these circuits become obvious.**

I. Introduction

WITHIN the past twenty years there have been many systems and techniques proposed for the measurement of phase in the UHF and microwave range of frequencies. In two recent papers Sparks presented a review and comparison of several of these systems and an excellent bibliography [1], [2]. In general, however, the literature is specialized and widely scattered.

Our purpose here is to present a theoretical analysis of bridge circuits suitable for the measurement of phase at these UHF and microwave frequencies. This analysis will lead to a classification for these circuits which clearly delineates the relationships between them. In an attempt to provide a unifying treatment the literature is extensively referenced.

The analysis is intended to be limited to circuits suitable for use at UHF frequencies and above, although it will be obvious that with present day components many of them can be used at lower frequencies. These circuits or systems are concerned with the measurement of phase of CW signals or signals with controlled repetitive modulation. Specialized techniques, for example such as those that might be required for unknown or uncooperative pulse modulation, are not considered.

II. Classification of Measurement Systems

The measurement of the relative phase of an RF signal involves either a comparison between the phase of an unknown signal and that of a reference signal, or a more fundamental measurement that involves the measurement of the changing character of the unknown signal with time. We are here concerned primarily with the former and will identify systems suitable for this comparison as belonging to one of several classes or subclasses.

Manuscript received December 29, 1965; revised May 16, 1966. This work was performed under sponsorship of the AF Avionics Laboratory, Contract AF 33(615)3216.
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